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Singular Optima in Geometrical Optimization of Structures

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Introduction

A POINT is said to be a local (relative) minimum if it has the least function value in its neighborhood but not necessarily the least function value for all the feasible region. In this Note, a point is said to be a singular optimum if it has the least function value in its neighborhood, but this neighborhood is a reduced (degenerate) feasible region, formed by assuming certain variables as zero. Singular optima are usually associated with changes in the topology of the structure. If the optimal solution is a singular point in the design space, it might be difficult or even impossible to arrive at the true optimum by numerical search algorithms. The singularity of the optimal topology in cross-sectional optimization of truss structures was first shown by Sved and Ginos.[1] Singular optima of grillages[2,3] and some properties of singular optimal topologies[4,5] were studied later. In particular, problems of continuous constraint functions have been discussed, and limiting stresses obtained in cases of elimination of members have been defined.[5]

In this Note, the effect of some preassigned parameters on singular optima in cross-sectional optimization is first demonstrated. Then, singular optima that might occur in geometrical optimization are presented. In simultaneous optimization of geometrical and cross-sectional variables, it is possible that some joints tend to coalesce during the solution process. It will be shown that under certain circumstances, reduced optimal structures obtained by elimination of members that coincide with others due to coalescence of joints, might represent singular optima that cannot be reached by simple numerical optimization.

Effect of Geometrical Preassigned Parameters

The following notation will be used for the various optimum design points: G is the global optimum, L local optimum, GS global singular optimum, and LS local singular optimum. To illustrate singular and local optima in the optimization of cross-sectional variables, consider the three beam grillage shown in Fig. 1 and subjected to two concentrated loads $P = 1$ at the intersections (arbitrary dimensions are assumed in all examples). Assume constant depth of 2.45 and the width of the cross sections for the longitudinal and transverse beams, denoted as X_1 and X_2, respectively, as design variables. The allowable stress is $\sigma^U = 10$, and the member's lengths are $\ell_x = 10.0$ and $\ell_y = 14.0$. Neglecting torsional rigidity of the elements, assuming only stress constraints and the volume of material as an objective function, the design space is shown in Fig. 2a and the solution process converges to point A. However, since $X_2 = 0$ at this point, the two transverse members and the constraint $\sigma_2 \leq \sigma^U$ are eliminated. The resulting global singular optimum is at point $GS(X_1 = 1.0, X_2 = 0, Z = 73.5)$ and not at point $A(X_1 = 1.275, X_2 = 0, Z = 93.7)$. It can be observed that the line segment A–GS is a reduced (degenerate) part of the feasible region. In addition, both points A and GS represent a topology of only the longitudinal beam. Although the optimum is a singular point in the design space, there is a single feasible region. Two or more separated regions might exist if upper or lower limits on design variables are considered.[5]

It is instructive to note that modifications in the objective function coefficients might result in different points of convergence and optima. Assuming, for example, the objective function

$$Z = 2.45(TX_1 + 56X_2) \rightarrow \min \qquad (1)$$

where T is a predetermined coefficient, then the various possible solutions are summarized in Table 1. It can be seen that convergence to the true optimum is achieved only if $T \geq 39.2$. It should be emphasized that singular optima exist also in alternative formulations of the problem,[6] such as simultaneous analysis and design (SAND). However, such optima do not exist in optimal plastic design, where the compatibility conditions are neglected.[6]

It has been noted[6] that changes in various preassigned parameters (such as geometrical parameters, allowable stresses, and limits on design variables and loadings) might result in singular optima. To illustrate the effect of geometrical parameters, assume various geometries for the grillage of Fig. 1. Results for $\ell_y = 14.0$, $\ell_y = 60.0$, and $\ell_y = 6.0$ are shown in Fig. 2 and in Table 2. As noted earlier, for $\ell_y = 14.0$ (Fig. 2a) the singular global optimum is at point GS. For $\ell_y = 60.0$ (Fig. 2b) the optimum is at point G, and for $\ell_y = 6.0$ (Fig. 2c) the optimum is at point G, both being nonsingular global optima.

Table 1 Effect of objective function coefficients, grillage, Fig. 1

Value of T	Point of convergence	True optimum
$T < 30.75$	A	GS
$T = 30.75$	Along A–B	GS
$30.75 < T < 39.2$	B	GS
$T = 39.2$	B	B and GS
$T > 39.2$	B	B

Table 2 Effect of geometry, grillage, Fig. 1

Figure	ℓ_y	Point	X_1	X_2	Z
2a	14	A	1.275	0	93.7
		GS	1.0	0	73.5
2b	60	G	1.0	0	73.5
		B	0	43.2	25396
2c	6	A	69.4	0	5103
		G	0	3.0	176.4

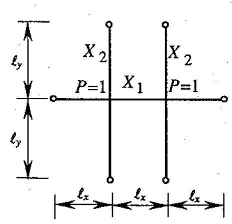

Fig. 1 Grillage.

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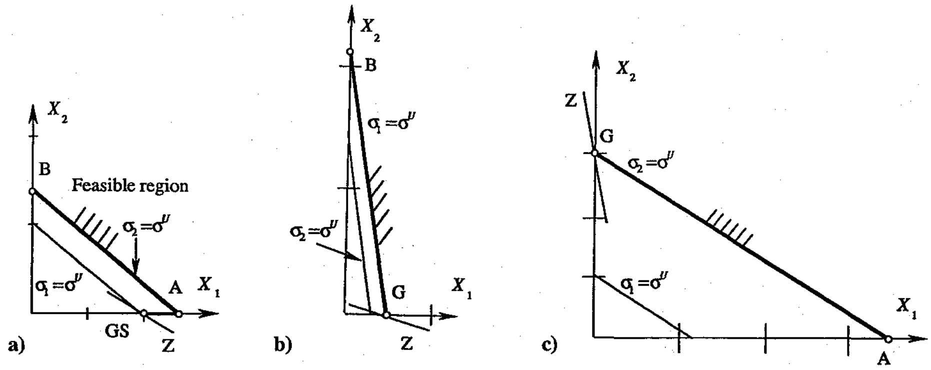

Fig. 2 Design space, effect of geometrical parameters.

Table 3 Optimum points 11-bar truss, Fig. 3

Case	Point	Y	Z
1	A	0	42.0
$H = 1.00$	G	1.0	30.0
	B	3.0	40.0
	LS	3.0	38.0
2	A	0	34.0
$H = 4.24$	L	1.0	31.1
	B	3.0	42.4
	GS	3.0	17.0

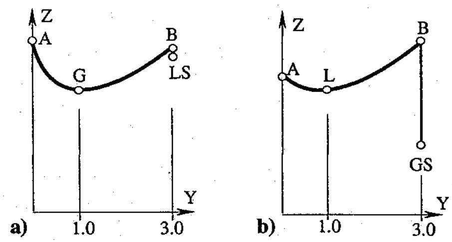

Fig. 4 Z vs Y, 11-bar truss: a) H = 1.0 and b) H = 4.24.

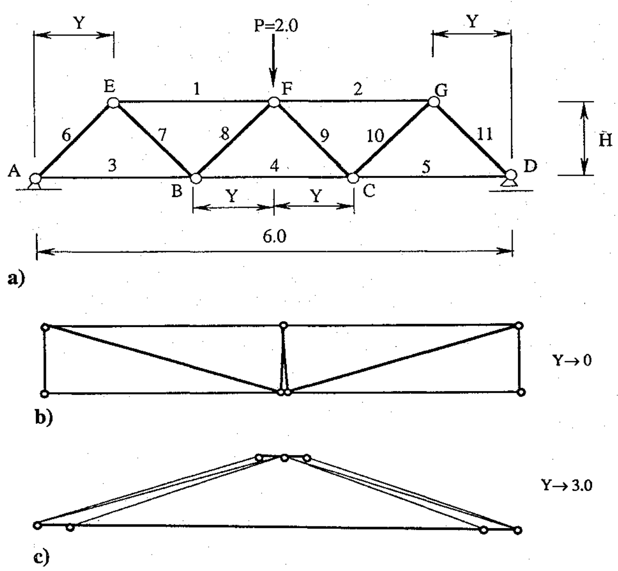

Fig. 3 Various geometries, 11-bar truss.

Singular Optima in Geometrical Optimization

To illustrate the phenomenon of singular optima in geometrical optimization, where two or more joints coalesce, consider the symmetric 11-bar truss shown in Fig. 3a and subjected to a single load $P = 2.0$. Assume the single geometrical variable Y, describing the horizontal location of joints B, C, E, and G. The objective function represents the volume of material and the constraints are related to stress and limitations on Y. Since the truss is a statically determinate structure, the optimal design will be fully stressed. Assuming $\sigma^U = -\sigma^L = 1.0$, then the optimal cross-sectional areas are given by $X_i = |f_i(Y)|$, where the members' forces $f_i(Y)$ are functions of Y. The problem can be stated in terms of only Y as

$$Z = \sum_{i=1}^{11} |f_i(Y)|\, \ell_i(Y) \rightarrow \min \tag{2}$$

$$0 \leq Y \leq 3.0 \tag{3}$$

where the length of the ith member ℓ_i is also a function of Y. The initial topology shown in Fig. 3a (for $Y = 1.0$), is changed for $Y = 0$ and for $Y = 3.0$ (Figs. 3b and 3c, respectively). Variation of Z with Y for two given depths ($H = 1.0$ and the optimal depth $H = 4.24$) are shown in Fig. 4 and the optimum points are summarized in Table 3. The following observations can be made:

1) For $H = 1.0$ (Fig. 4a), the global optimum is at point $G(Y = 1.0)$ and there is a local singular optimum at point $LS(Y = 3.0)$.

2) For the optimal depth $H = 4.24$ (Fig. 4b), the global singular optimum is at point $GS(Y = 3.0, Z = 17.0)$. However, the local optimum reached by optimizing Y is at point $L(Y = 1.0, Z = 31.1)$. That is, the solution reached by numerical optimization of Y is heavier than the global optimum by 83%!

The singular optima at $Y = 3.0$ are the result of changes in the topology of the structure. Specifically, as Y approaches 3.0, the joints A-B, C-D, E-F-G, and the members 6-7-8, 9-10-11 tend to coalesce. Just before that, the forces in members 6–11 have the same magnitude but the forces in members 7 and 10 are in the opposite direction. At the limit ($Y = 3.0$) the three members 6, 7, 8 (and 9, 10, 11) coalesce and become a single member. The weight of the diagonal members is reduced by 2/3 and the singular global optimum is at point GS (Fig. 4b).

Concluding Remarks

The major difficulty in problems having singular and local optima is that the solution process might converge to a nonoptimal design. It has been noted that such optima might exist in structural optimization problems where either cross-sectional variables or geometrical variables are optimized. The type of the optimum depends on the chosen design variables and on the selected preassigned parameters.

Singular optima are usually associated with changes in the topology. It is shown that in geometrical optimization such optima might exist even in simple statically determinate structures, where joints and members coalesce. If the optimal solution is a singular point in the design space, it might be difficult or even impossible to arrive at the true optimum by numerical search algorithms.

Singular optima are independent of the problem formulation and exist either in the common formulation in the design variables space or in the SAND formulation. To overcome the difficulty involved in the solution process it might be necessary to adopt new layout optimization approaches, where members and joints are added to an initial reduced structure. Such an approach is the subject of a future article.[7]

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Errata

Observations on Using Experimental Data as Boundary Conditions for Computations

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DURING production of this paper, a measurement was incorrectly stated. AIAA regrets the error.

Page 177

In the paragraph that begins "However, the zero normal velocity . . . ," the third sentence should read as follows:

Both sets of results were obtained using the standard Baldwin–Lomax turbulence model and a computational grid with a minimum y^+ of 4.4.